SABINA THz FEL

Beam Distribution Analysis

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REFERENCE PERIODIC SOLUTION AT THE UNDULATOR ENTRANCE

$$k_u = \sqrt{h}\,\frac{\pi K}{\lambda_u \gamma}$$

Beam energy = 100.000 MeV

gamma = 195.69471624

Lu = 1.32 [m] undulator length

Lg = 0.392 [m] gap between undulators

lambda_u = 0.055

K_u = 3.4

Cell=Undulator+Drift

$$L_{cell} = L_u + L_d$$

Transport matrix of one cell:

$$M_{cell} = M_{drift}\, M_{und}$$

Periodic Twiss condition:

$$M_{cell} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

Periodic Twiss at UND entrance:

X: alpha = -0.31449038, beta = 0.68487063

Y: alpha = -0.31449038, beta = 0.68487063

Conditions

- Reference Periodic Solution at the Undulator Entrance
- Matching beam Parameter ≤ 1.05

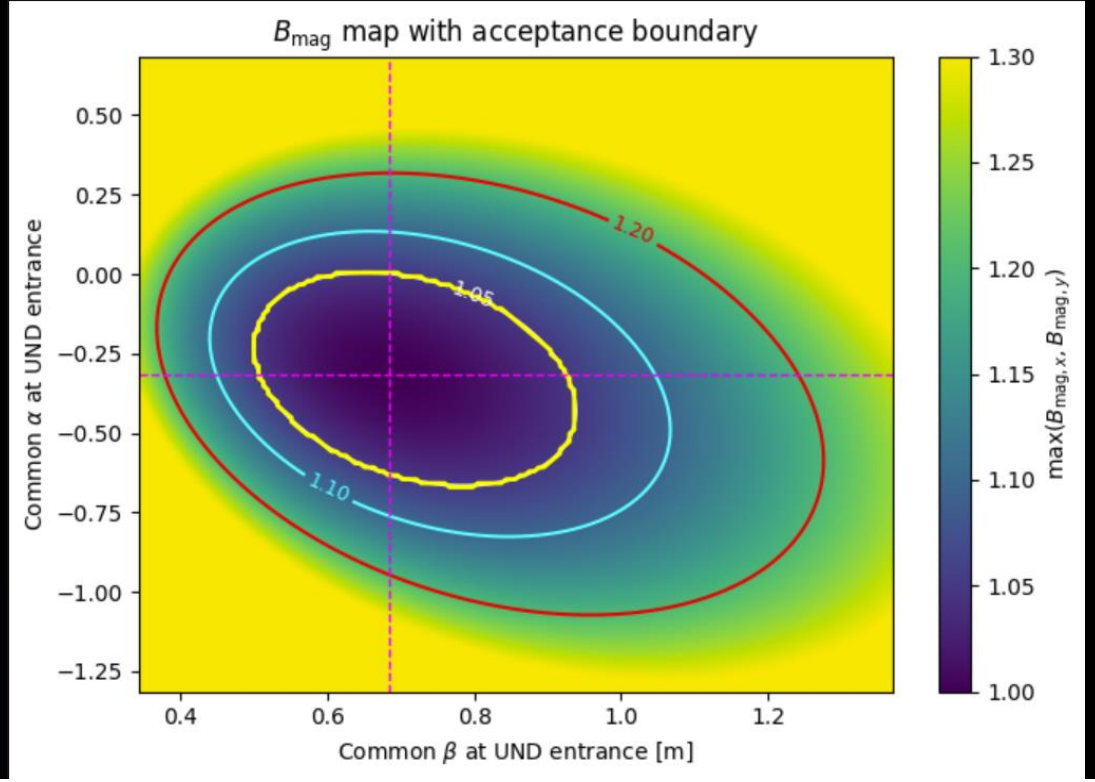
$$B_{mag} = \frac{1}{2}\left(\frac{\beta}{\beta_0} + \frac{\beta_0}{\beta} + \frac{(\alpha - \alpha_0)^2}{\beta\beta_0}\right)$$

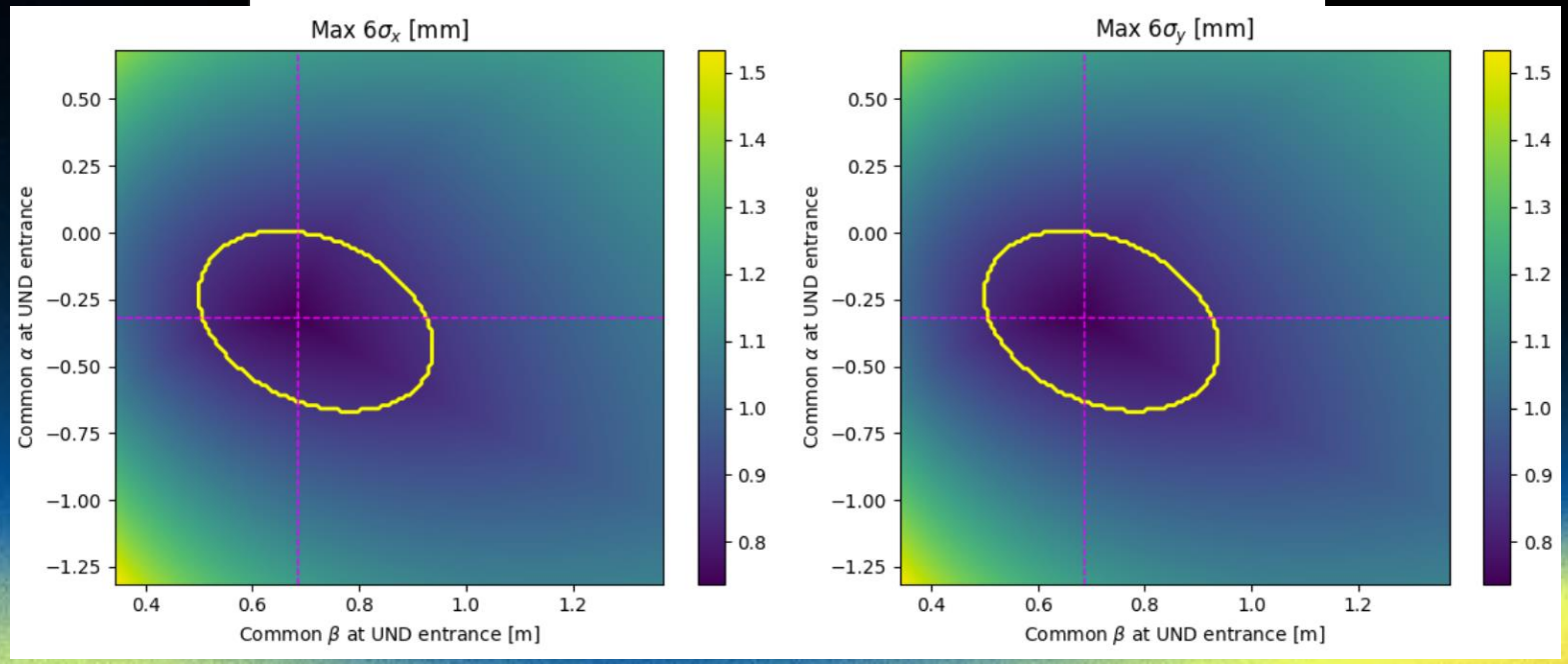
- Must remain below the vacuum chamber

6σx, 6σy ≤ vacuum chamber half-aperture

vacuum chamber half-aperture = 4mm

- Normalized emittance upper limit = 3.0 [mm-mrad]


B_{mag} map with acceptance boundary
1.20
1.05
1.10
Common α at UND entrance
Common β at UND entrance [m]
max($B_{mag,x}$, $B_{mag,y}$)


Max 6σ_x [mm]
Max 6σ_y [mm]
Common α at UND entrance
Common β at UND entrance [m]

Accepted Range

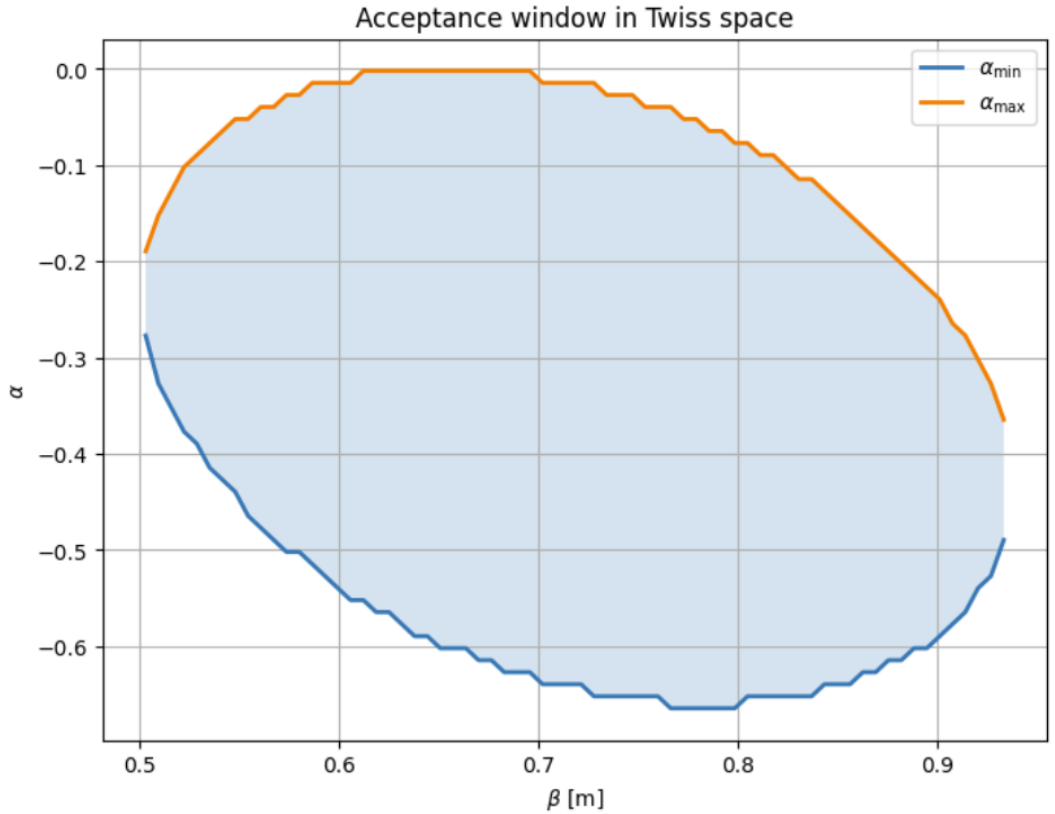
Accepted alpha range:

-0.6644903752840433 → -0.0019903752840433 3

Accepted beta range:

0.5029518712295972 → 0.9331362376855505

beta-dependent alpha ranges

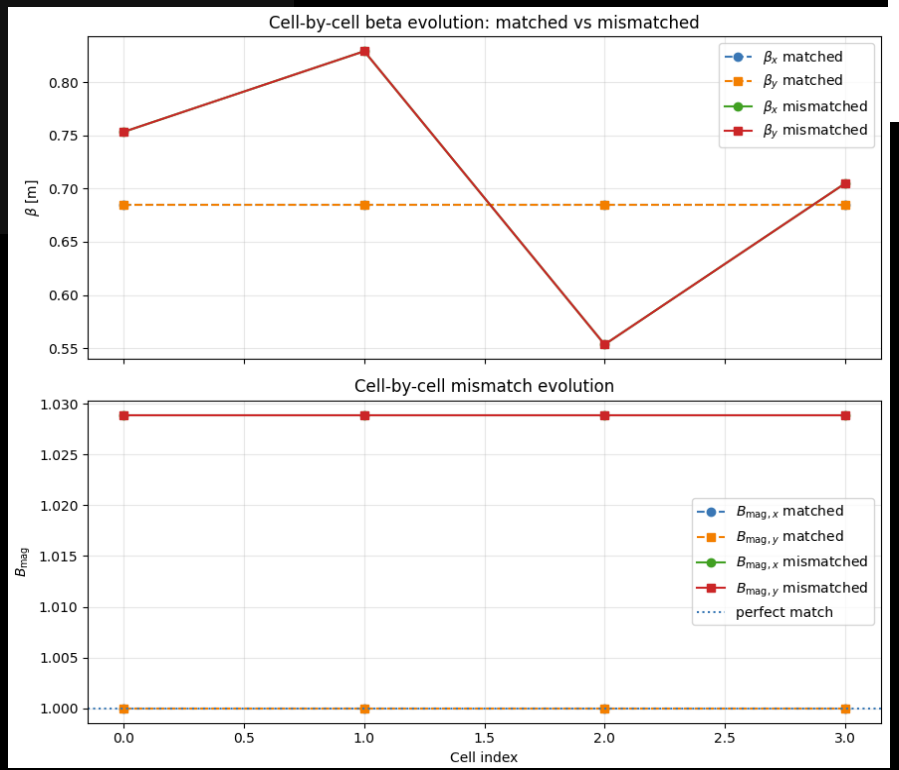


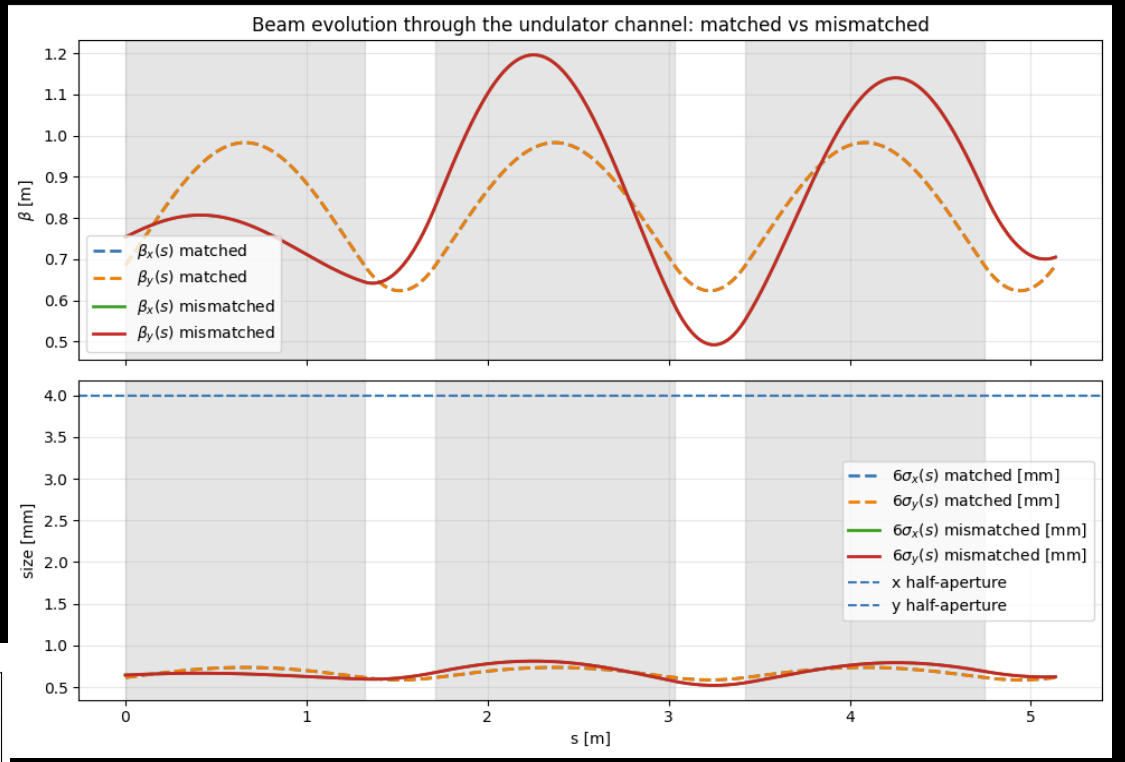
EXAMPLE

```
========== INPUT TWISS ==========
Matched case   : alpha = -0.314490, beta = 0.684871
Mismatched case: alpha = -0.114490, beta = 0.753358

========== MATCHED CASE ==========
accepted     = True
mismatch_ok  = True
aperture_ok  = True
bmag_x_max   = 1.0
bmag_y_max   = 1.0
6sigma_x_max = 0.736552 mm
6sigma_y_max = 0.736552 mm

========== MISMATCHED CASE ==========
accepted     = True
mismatch_ok  = True
aperture_ok  = True
bmag_x_max   = 1.0288948440785568
bmag_y_max   = 1.0288948440785568
6sigma_x_max = 0.812451 mm
6sigma_y_max = 0.812451 mm
```





G O A L : Find Q1 Twiss configurations that can be matched into the accepted UND region.

Model consistency check for trusted seeds:

Gill's beam:

Q1 Twiss = (ax, bx, ay, by) = (-3.416024, 46.854790, -3.393246, 46.597587)

resid_norm = 8.935e-14

Bmag(screen) = (1.000000, 1.000000)

Bmag(und) = (1.000000, 1.000000)

Optimised Anna's beam:

Q1 Twiss = (ax, bx, ay, by) = (-54.274076, 37.022950, 85.492919, 12.134732)

resid_norm = 6.583e-13

Bmag(screen) = (1.000000, 1.000000)

Bmag(und) = (1.000000, 1.000000)

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model_benchmark:

Q1 Twiss = (ax, bx, ay, by) = (-107.055171, 60.328707, 1.786450, 0.051795)

resid_norm = 1.528e-12

Bmag(screen) = (1.000000, 1.000000)

Bmag(und) = (1.000000, 1.000000)

Local scan boxes:

Seed: Gill's beam
alpha_x: -5.416024058979684 -> -1.4160240589796844
beta_x : 42.16931130148905 -> 51.540269368486626
alpha_y: -5.393246198278122 -> -1.3932461982781228
beta_y : 41.93782855903557 -> 51.25734601659904

Seed: Optimised Anna's beam
alpha_x: -59.701483571096446 -> -48.84666837635163
beta_x : 33.32065544503011 -> 40.725245543925695
alpha_y: 76.943626677243 -> 94.042210383297
beta_y : 10.13473215605042 -> 14.13473215605042

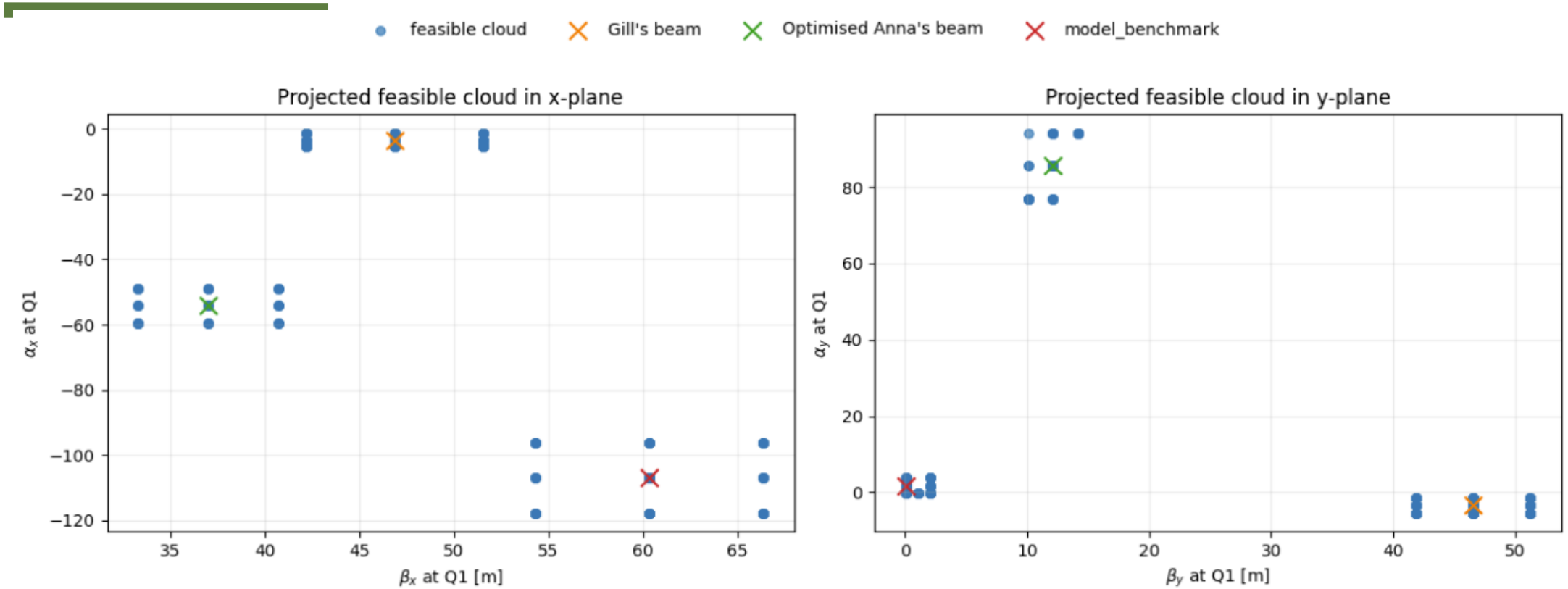
Seed: model_benchmark
alpha_x: -117.76068860134892 -> -96.34965431019457
beta_x : 54.29583668648195 -> 66.36157817236683
alpha_y: -0.2135498069448749 -> 3.786450193055125
beta_y : 0.05 -> 2.0517952479956545

Total explored points = 243

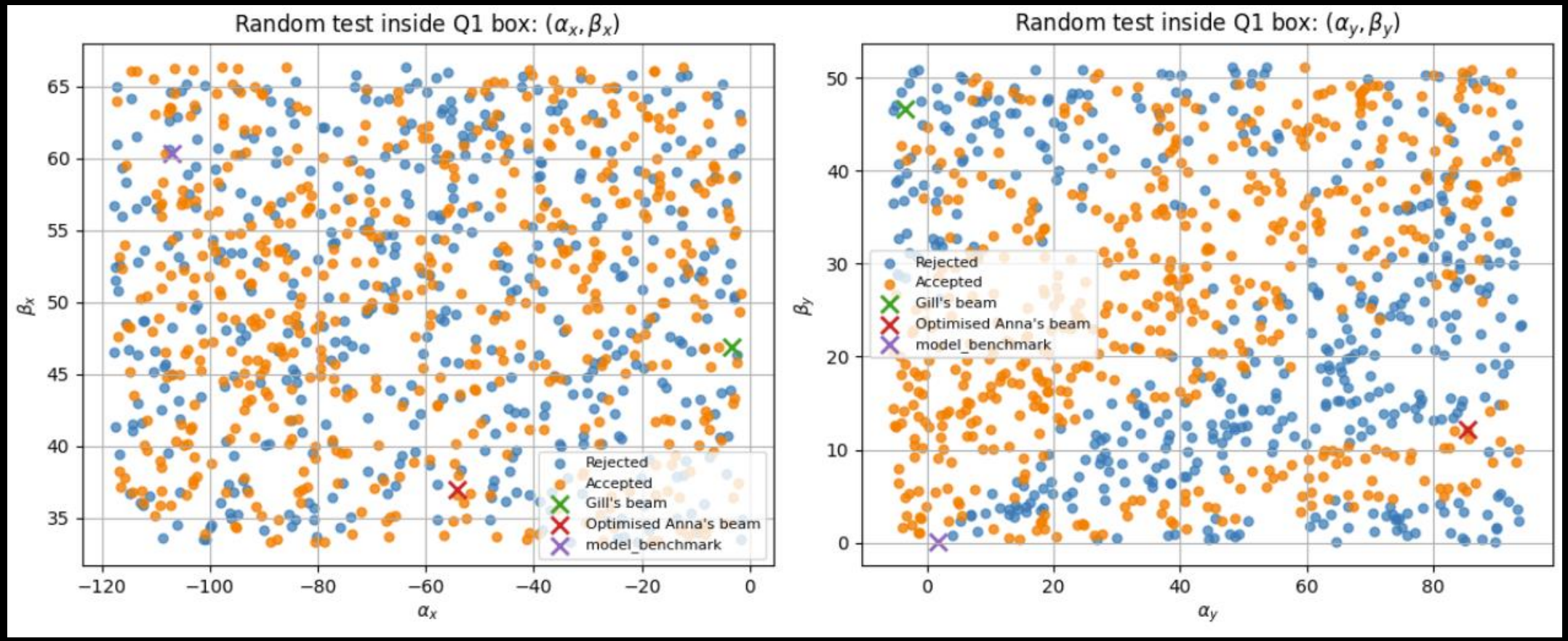
Feasible cloud points = 188

Feasible cloud at Q1

alpha_x : -117.760689 -> -1.416024
beta_x : 33.320655 -> 66.361578
alpha_y : -5.393246 -> 94.042210
beta_y : 0.050000 -> 51.257346



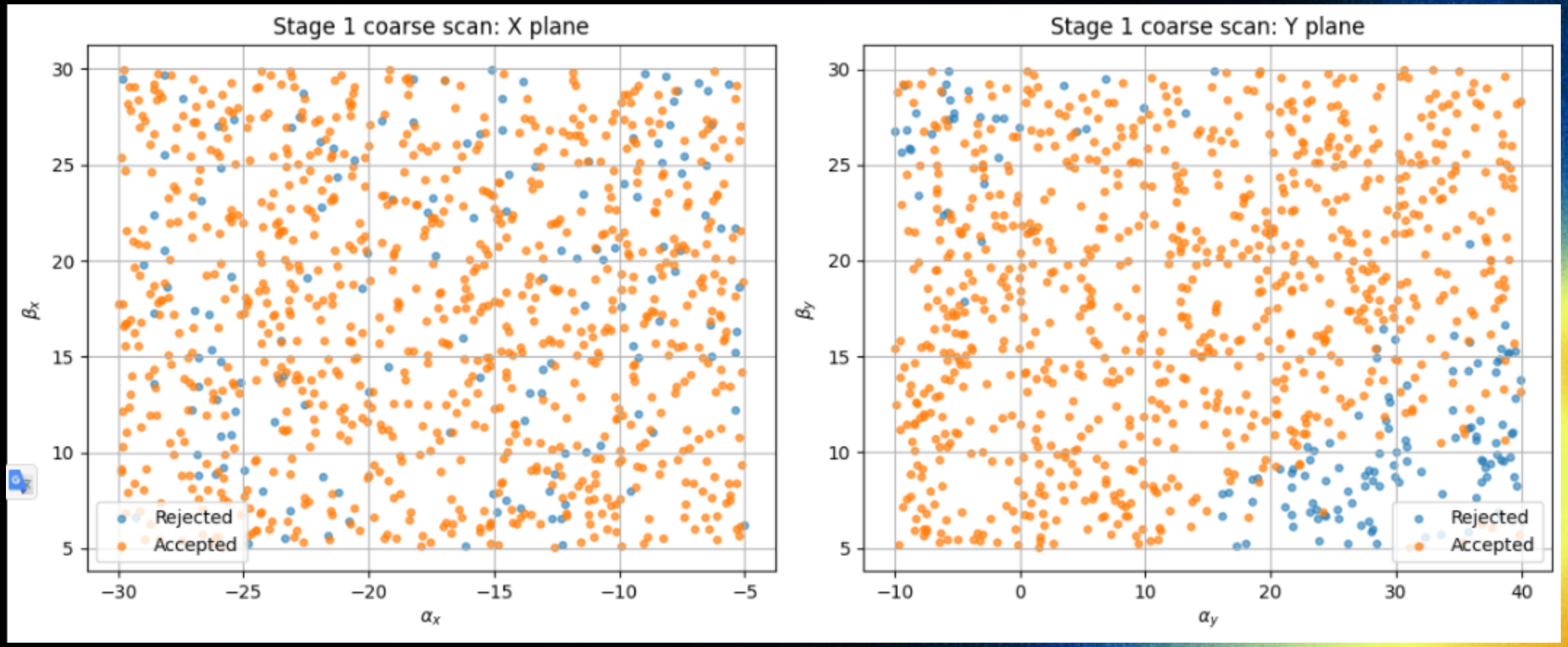
Total tested = 1000 Accepted = 540 Acceptance ratio ~ 54.00%



Key findings:
- Feasible region is not a box
- Working region exists (~54% success)
- High-probability zones (≥80%) identified in projections

Total tested = 1000 Accepted = 854 Acceptance ratio = 85.40%

Number of accepted points = 854
alpha_x : -29.996633 -> -5.084079
beta_x : 5.049803 -> 29.945521
alpha_y : -9.999418 -> 39.944631
beta_y : 5.036846 -> 29.992942



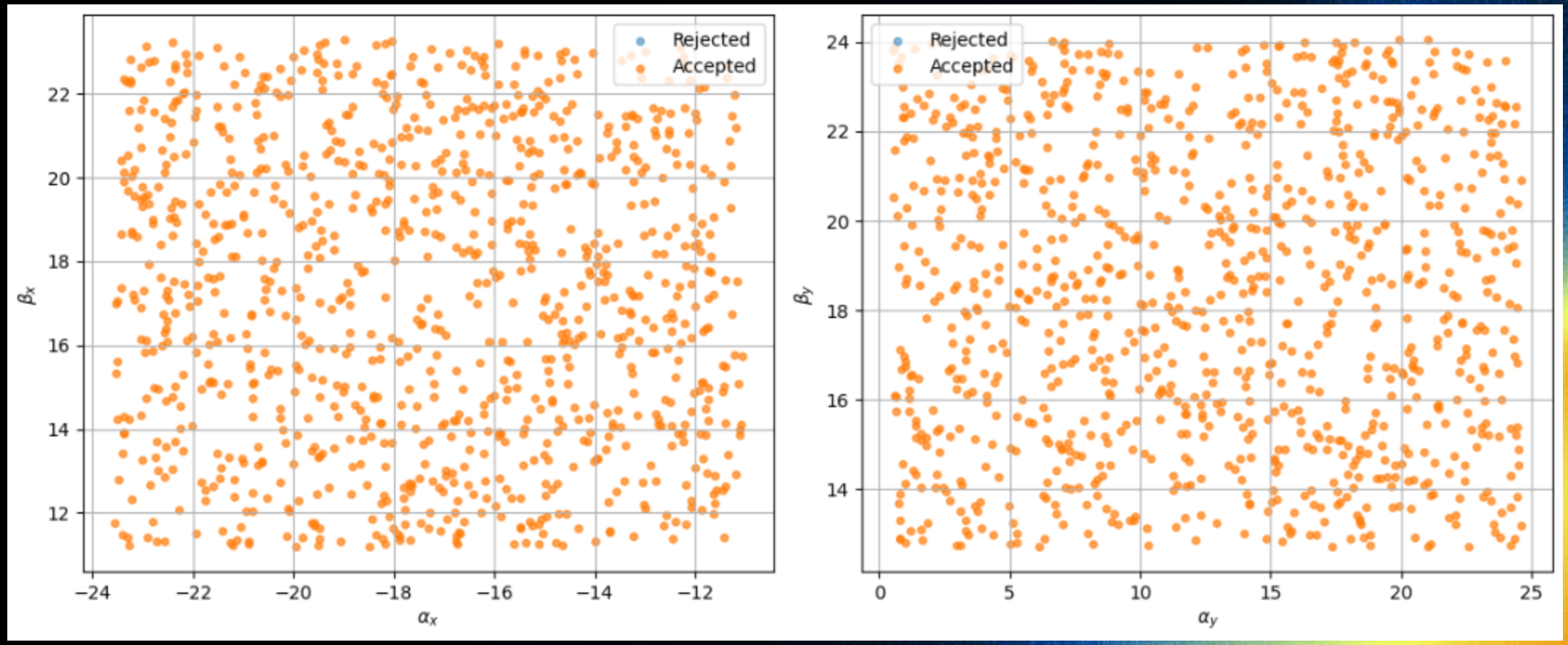
Refinement box :

alpha_x: -23.57954327597527 -> -11.049546462108948

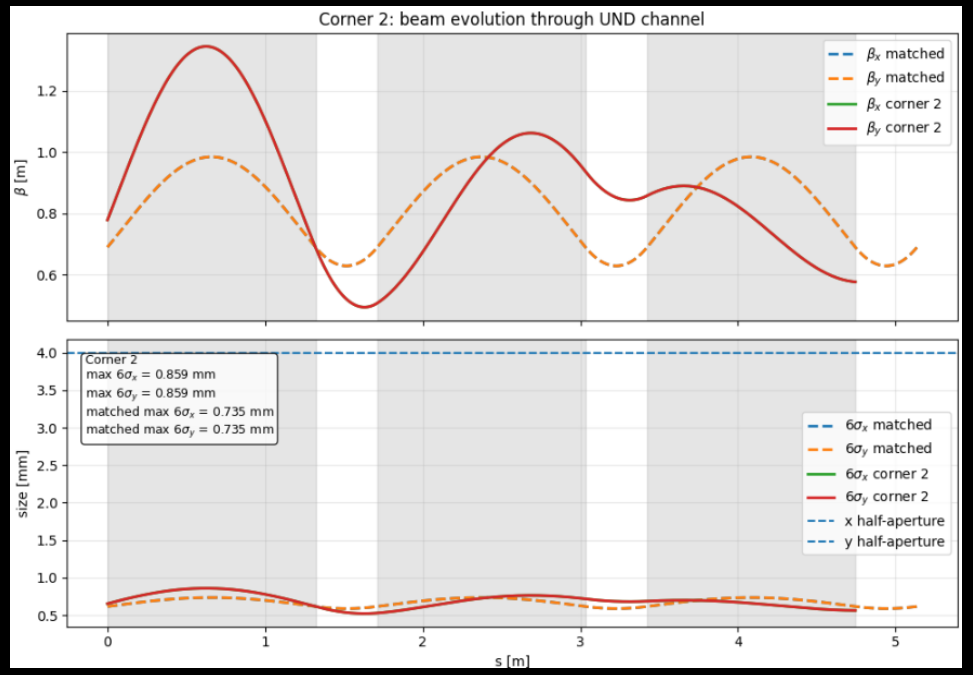
beta_x : 11.190064377130104 -> 23.289156597539737

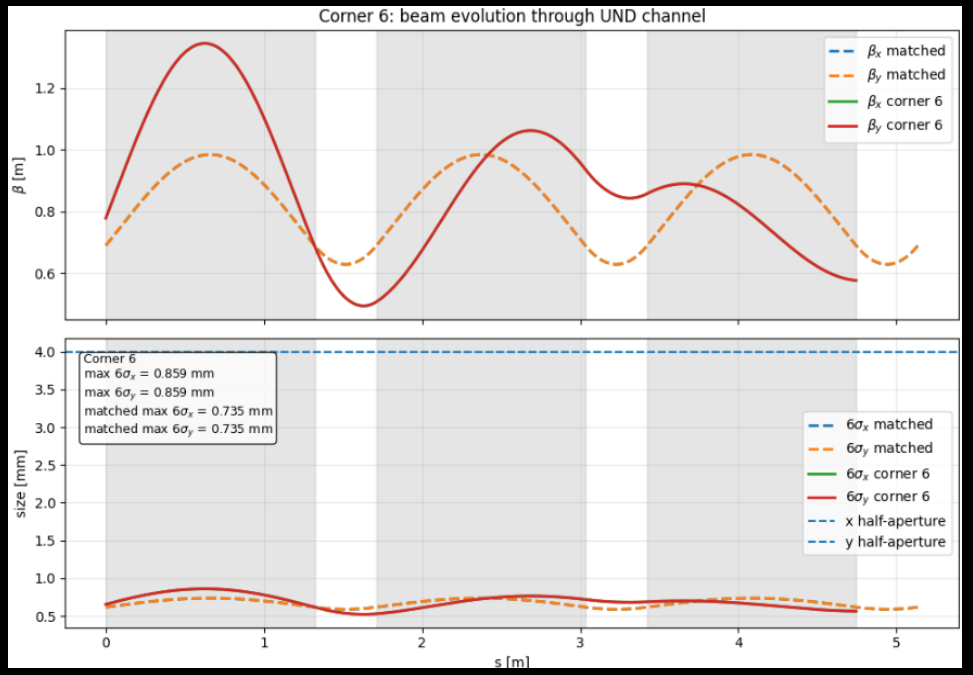
alpha_y: 0.5445362324769261 -> 24.610511868752653

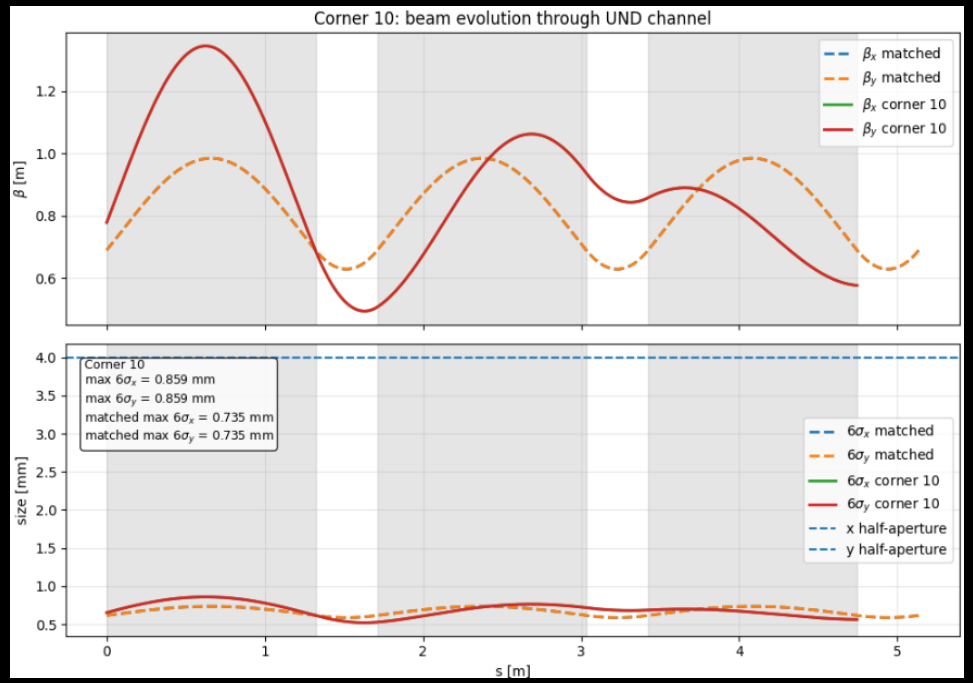
beta_y : 12.717359764570213 -> 24.05289008167352

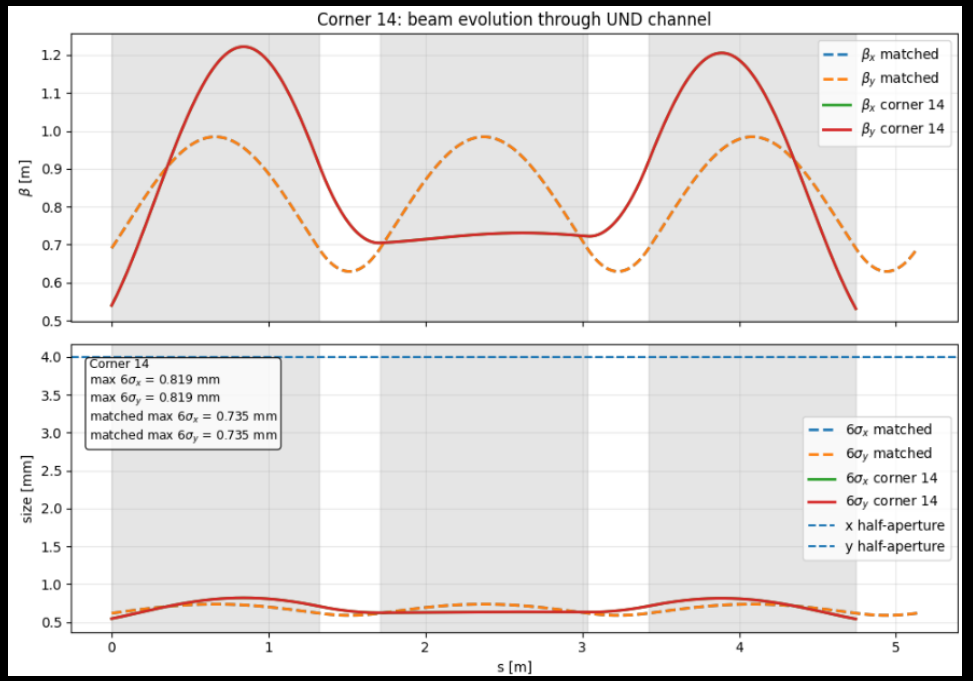


Corners index	αx Q1	βx Q1	αy Q1	βy Q1	α UND	β UND	Accepted	Max 6σ
0	-23.806	11.032	-0.031	12.897	-0.661	0.779	✅	0.859
1	-23.806	11.032	-0.031	24.492	-0.449	0.591	✅	0.815
2	-23.806	11.032	24.257	12.897	-0.661	0.779	✅	0.859
3	-23.806	11.032	24.257	24.492	-0.661	0.779	✅	0.859
4	-23.806	23.107	-0.031	12.897	-0.661	0.779	✅	0.859
5	-23.806	23.107	-0.031	24.492	-0.661	0.779	✅	0.859
6	-23.806	23.107	24.257	12.897	-0.661	0.779	✅	0.859
7	-23.806	23.107	24.257	24.492	-0.661	0.779	✅	0.859
8	-11.247	11.032	-0.031	12.897	-0.661	0.779	✅	0.859
9	-11.247	11.032	-0.031	24.492	-0.661	0.779	✅	0.859
10	-11.247	11.032	24.257	12.897	-0.661	0.779	✅	0.859
11	-11.247	11.032	24.257	24.492	-0.661	0.779	✅	0.859
12	-11.247	23.107	-0.031	12.897	-0.661	0.779	✅	0.859
13	-11.247	23.107	-0.031	24.492	-0.661	0.779	✅	0.859
14	-11.247	23.107	24.257	12.897	-0.399	0.539	✅	0.819
15	-11.247	23.107	24.257	24.492	-0.661	0.779	✅	0.859


Corner 2: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 2
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 2
βy corner 2
6σx matched
6σy matched
6σx corner 2
6σy corner 2
x half-aperture
y half-aperture


Corner 6: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 6
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 6
βy corner 6
6σx matched
6σy matched
6σx corner 6
6σy corner 6
x half-aperture
y half-aperture

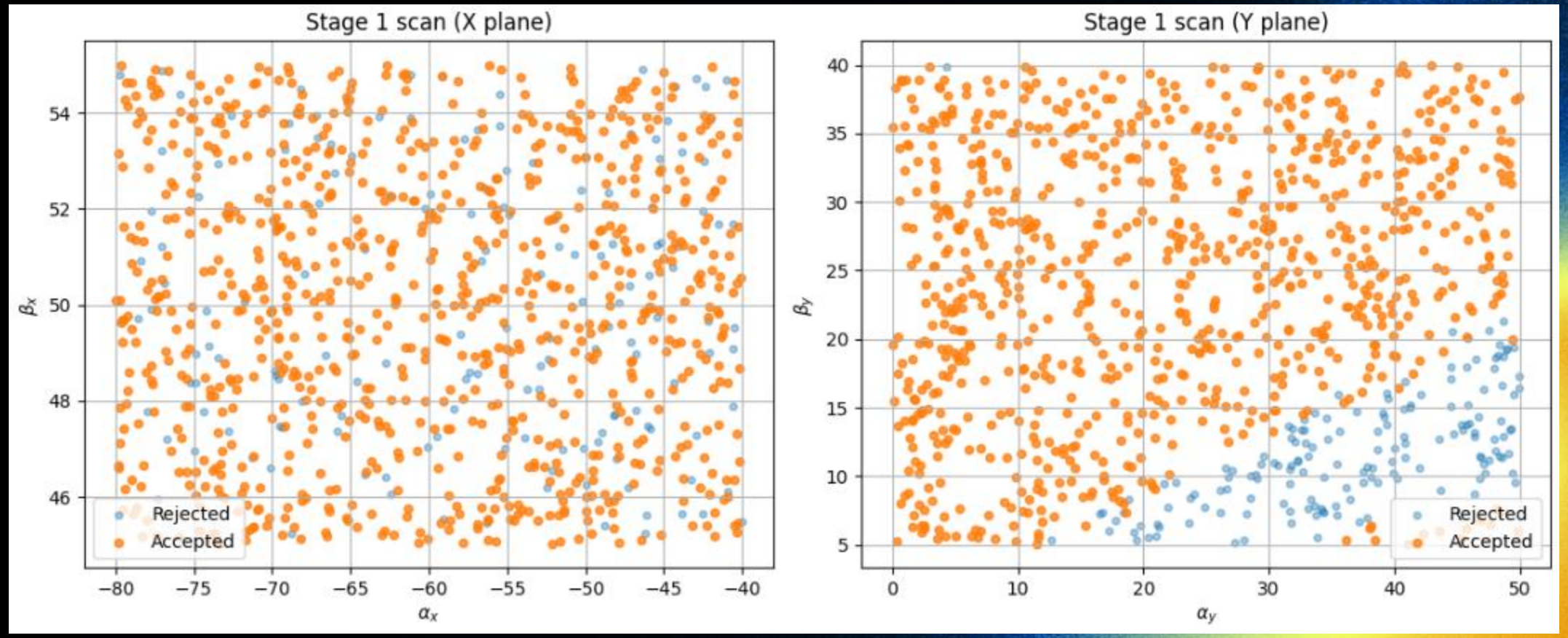

Corner 10: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 10
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 10
βy corner 10
6σx matched
6σy matched
6σx corner 10
6σy corner 10
x half-aperture
y half-aperture


Corner 14: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 14
max 6σx = 0.819 mm
max 6σy = 0.819 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 14
βy corner 14
6σx matched
6σy matched
6σx corner 14
6σy corner 14
x half-aperture
y half-aperture

Scale factor	Box enlargement	Failed corners	Surviving corners	Status
1.0	nominal safe box	0 / 16	16 / 16	fully safe
1.1	+10%	1 / 16	15 / 16	mostly stable
1.2	+20%	1 / 16	15 / 16	mostly stable
1.3	+30%	2 / 16	14 / 16	boundary degradation
1.4	+40%	4 / 16	12 / 16	no longer uniformly safe
1.5	+50%	4 / 16	12 / 16	no longer uniformly safe
1.6	+60%	4 / 16	12 / 16	no longer uniformly safe
1.7	+70%	4 / 16	12 / 16	no longer uniformly safe
1.8	+80%	4 / 16	12 / 16	no longer uniformly safe
1.9	+90%	5 / 16	11 / 16	degraded
2.0	+100%	6 / 16	10 / 16	degraded

Total tested = 1000 Accepted = 843 Acceptance ratio = 84.30%

alpha_x : -79.994612 -> -40.134527
beta_x : 45.019921 -> 54.995577
alpha_y : 0.000582 -> 49.917376
beta_y : 5.051584 -> 39.990119



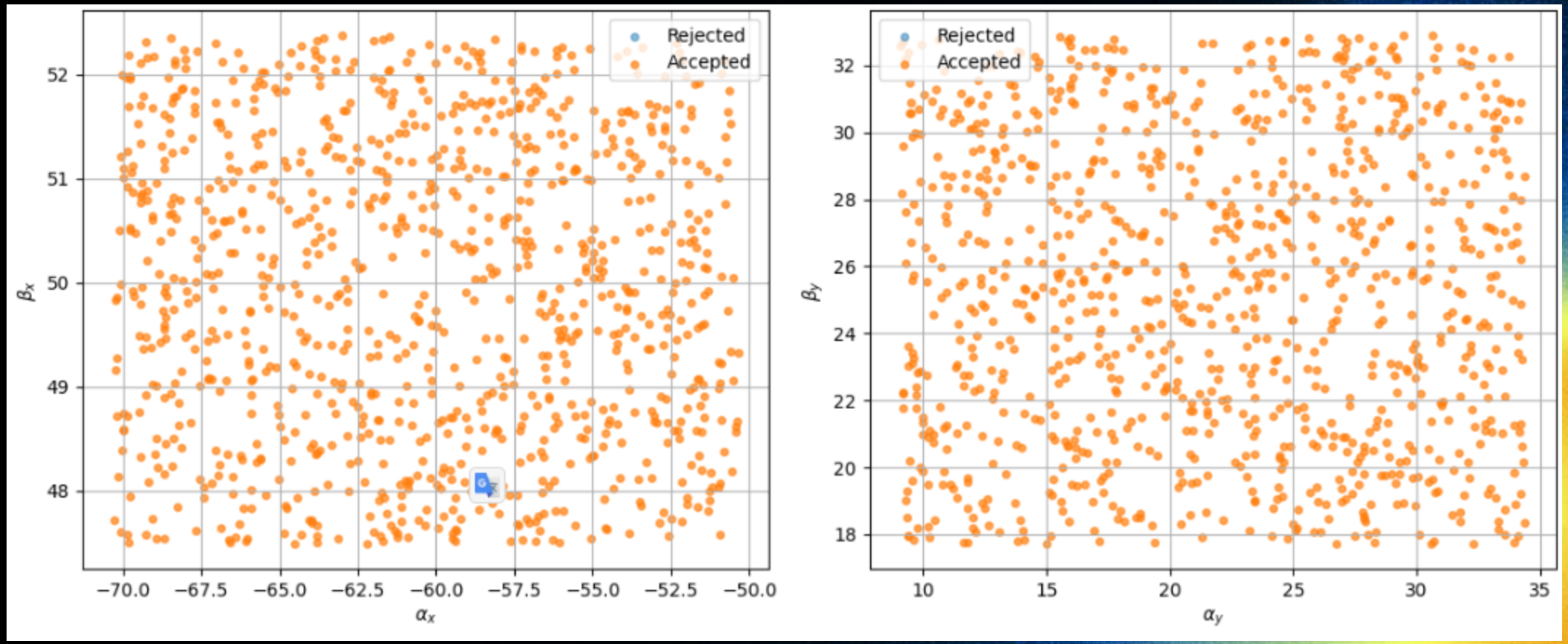
Refinement box :

alpha_x: -70.32046607643886 -> -50.352550821755315

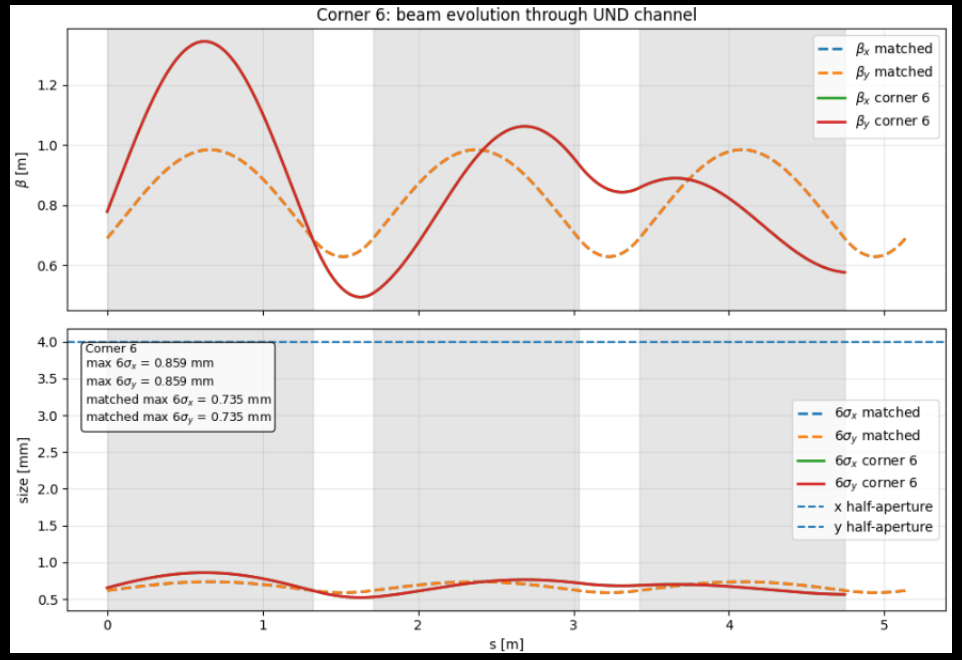
beta_x : 47.490032508682525 -> 52.3773960914233

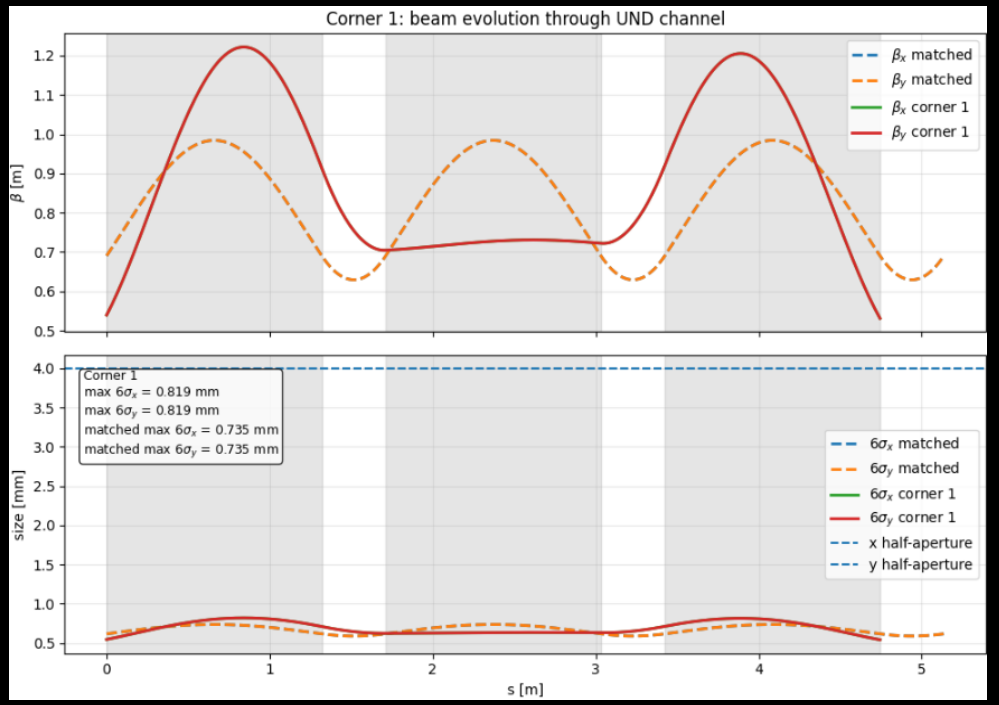
alpha_y: 9.098572072394997 -> 34.40311097145742

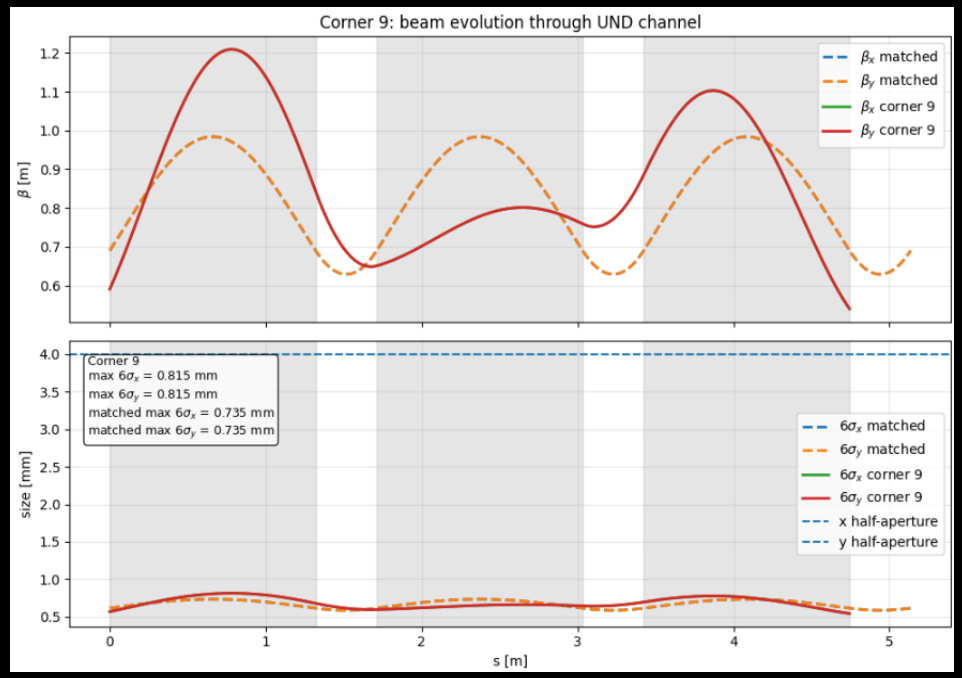
beta_y : 17.713842797256 37 -> 32.896502666292534

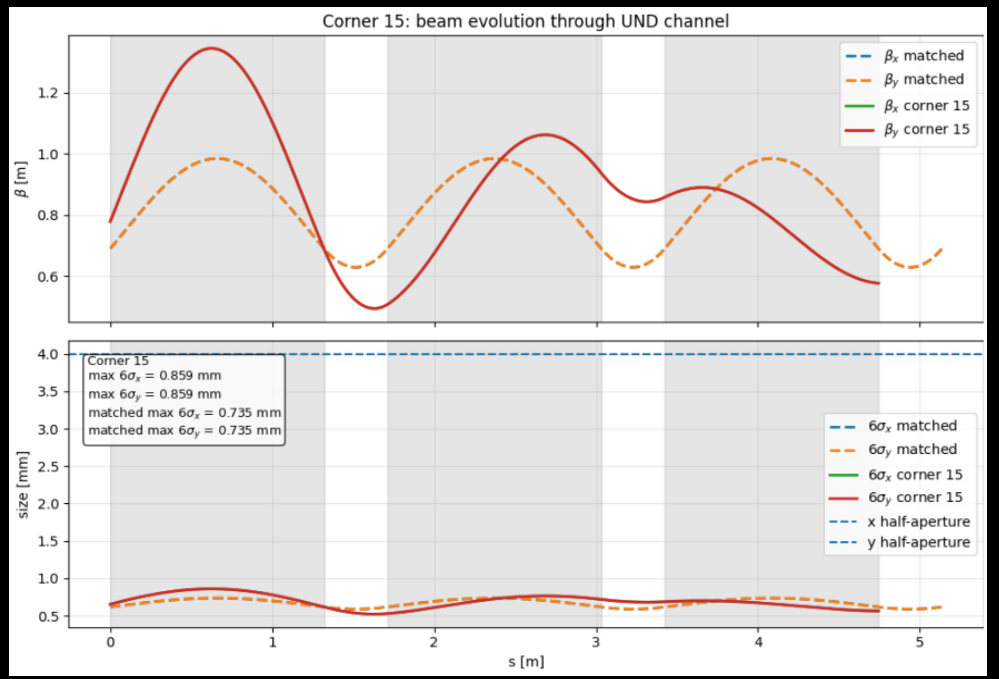


Corners index	αx Q1	βx Q1	αy Q1	βy Q1	α UND	β UND	Accepted	Max 6σ
0	-70.320	47.490	9.099	17.714	-0.661	0.779	✅	0.859
1	-70.320	47.490	9.099	32.897	-0.399	0.539	✅	0.819
2	-70.320	47.490	34.403	17.714	-0.661	0.779	✅	0.859
3	-70.320	47.490	34.403	32.897	-0.661	0.779	✅	0.859
4	-70.320	52.377	9.099	17.714	-0.661	0.779	✅	0.859
5	-70.320	52.377	9.099	32.897	-0.399	0.539	✅	0.819
6	-70.320	52.377	34.403	17.714	-0.661	0.779	✅	0.859
7	-70.320	52.377	34.403	32.897	-0.661	0.779	✅	0.859
8	-50.353	47.490	9.099	17.714	-0.661	0.779	✅	0.859
9	-50.353	47.490	9.099	32.897	-0.449	0.591	✅	0.815
10	-50.353	47.490	34.403	17.714	-0.661	0.779	✅	0.859
11	-50.353	47.490	34.403	32.897	-0.661	0.779	✅	0.859
12	-50.353	52.377	9.099	17.714	-0.661	0.779	✅	0.859
13	-50.353	52.377	9.099	32.897	-0.449	0.591	✅	0.815
14	-50.353	52.377	34.403	17.714	-0.661	0.779	✅	0.819
15	-50.353	52.377	34.403	32.897	-0.661	0.779	✅	0.859


Corner 6: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 6
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 6
βy corner 6
6σx matched
6σy matched
6σx corner 6
6σy corner 6
x half-aperture
y half-aperture


Corner 1: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 1
max 6σx = 0.819 mm
max 6σy = 0.819 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 1
βy corner 1
6σx matched
6σy matched
6σx corner 1
6σy corner 1
x half-aperture
y half-aperture


Corner 9: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 9
max 6σx = 0.815 mm
max 6σy = 0.815 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 9
βy corner 9
6σx matched
6σy matched
6σx corner 9
6σy corner 9
x half-aperture
y half-aperture

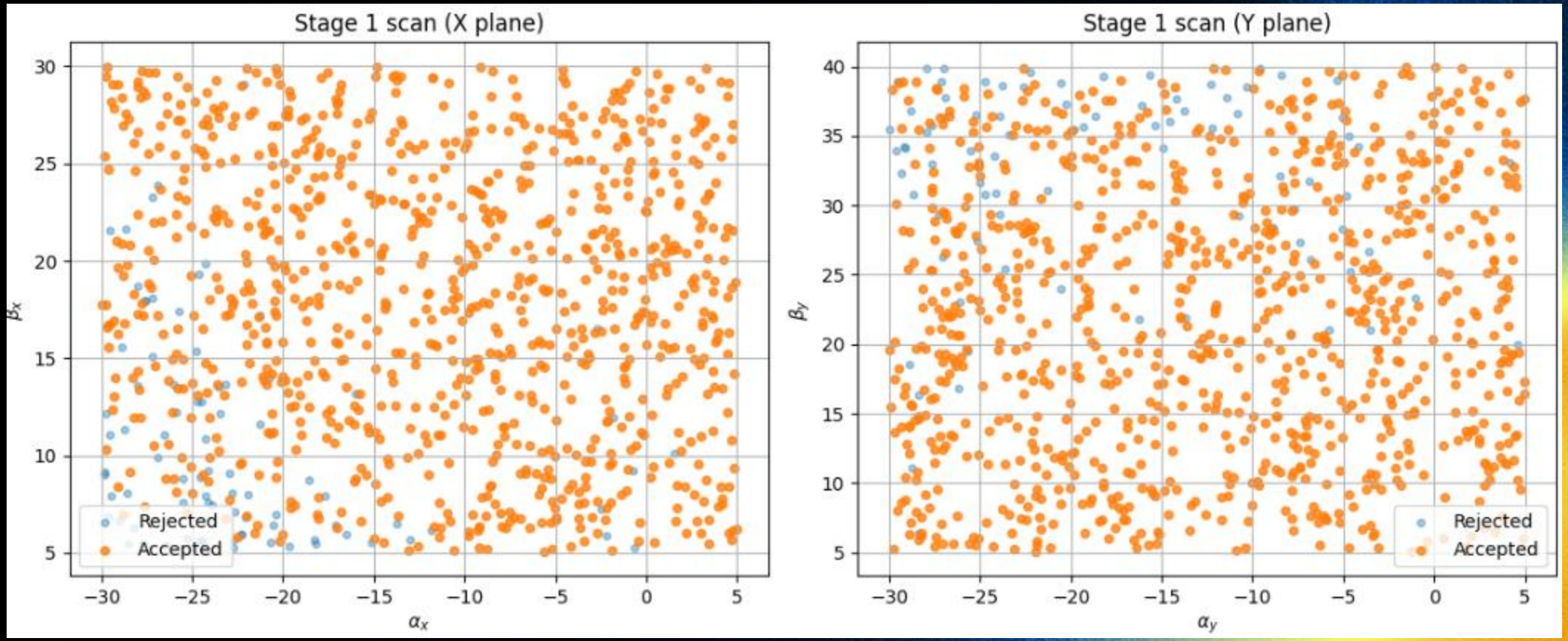

Corner 15: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 15
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 15
βy corner 15
6σx matched
6σy matched
6σx corner 15
6σy corner 15
x half-aperture
y half-aperture

$$x_{scaled} = x_{center} + s\,(x - x_{center})$$

Scale factor	Box enlargement	Failed corners	Surviving corners	Status
1.0	nominal safe box	0 / 16	16 / 16	fully safe
1.1	+10%	0/ 16	16 / 16	fully safe
1.2	+20%	2 / 16	14 / 16	boundary degradation
1.3	+30%	3 / 16	13/ 16	boundary degradation
1.4	+40%	4 / 16	12 / 16	no longer uniformly safe
1.5	+50%	4 / 16	12 / 16	no longer uniformly safe
1.6	+60%	5/ 16	11 / 16	degraded
1.7	+70%	6 / 16	10 / 16	degraded
1.8	+80%	6 / 16	10 / 16	degraded
1.9	+90%	6 / 16	10 / 16	degraded
2.0	+100%	6 / 16	10 / 16	degraded

Total tested = 1000 Accepted = 915 Acceptance ratio = 91.50%

alpha_x : -29.995286 -> 4.979480
beta_x : 5.080457 -> 29.988943
alpha_y : -29.999593 -> 4.961241
beta_y : 5.051584 -> 39.990119



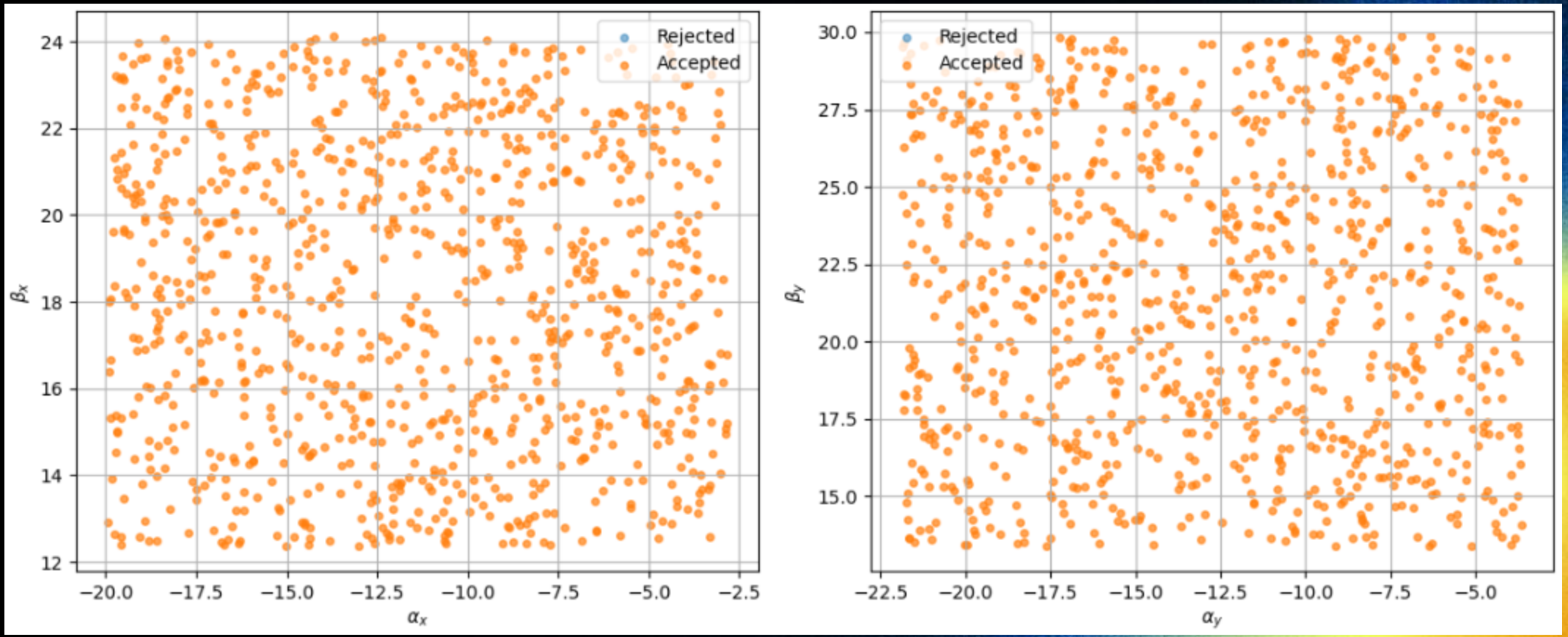
Refinement box :

alpha_x: -19.985137213402993 -> -2.813807187349086

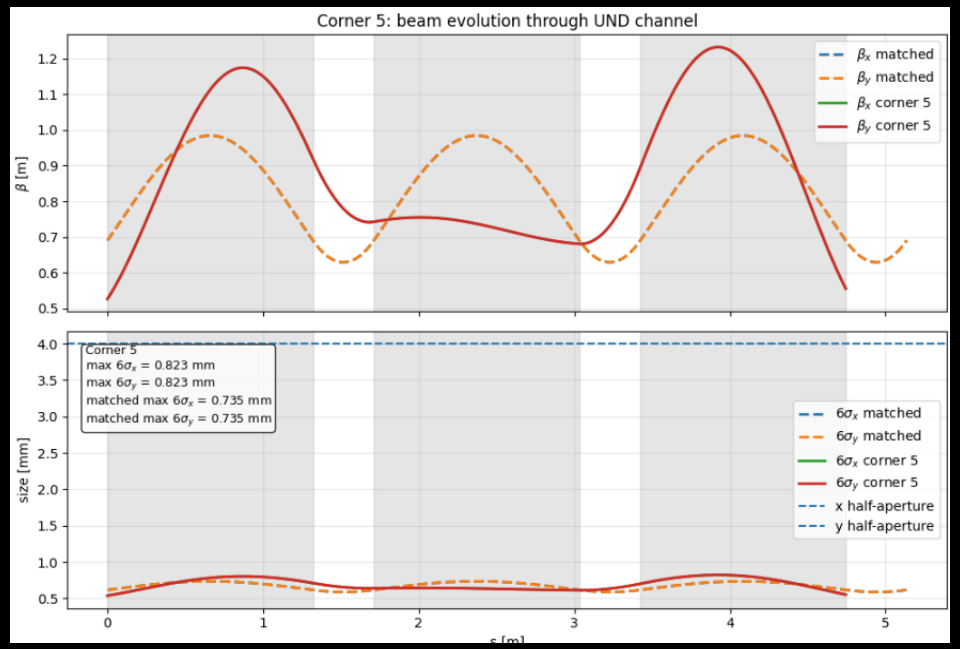
beta_x : 12.361029228905458 -> 24.121163280564048

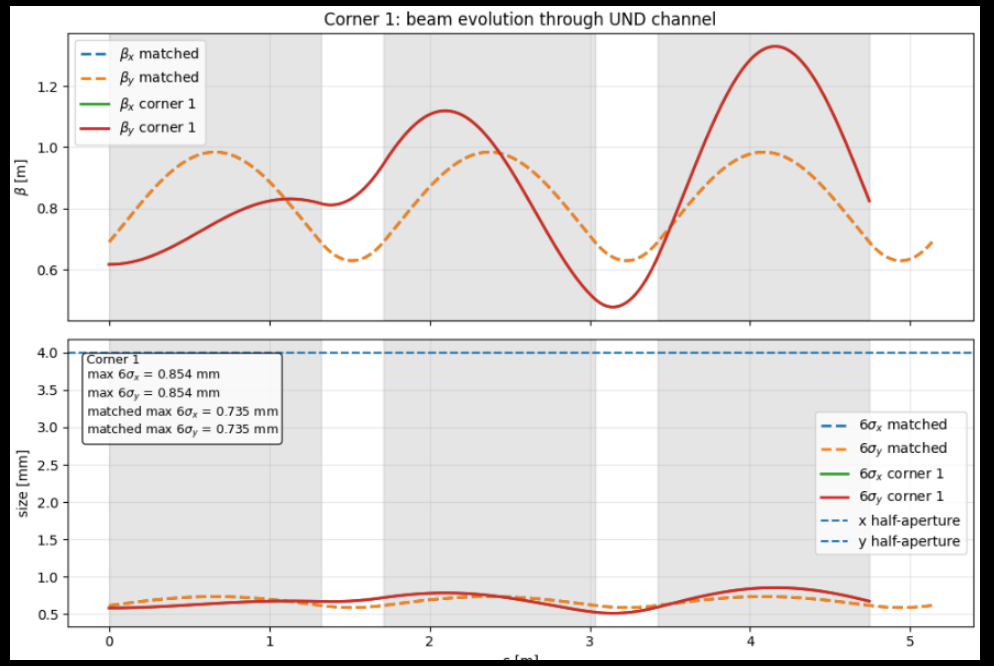
alpha_y: -21.872956191114973 -> -3.612975239632803

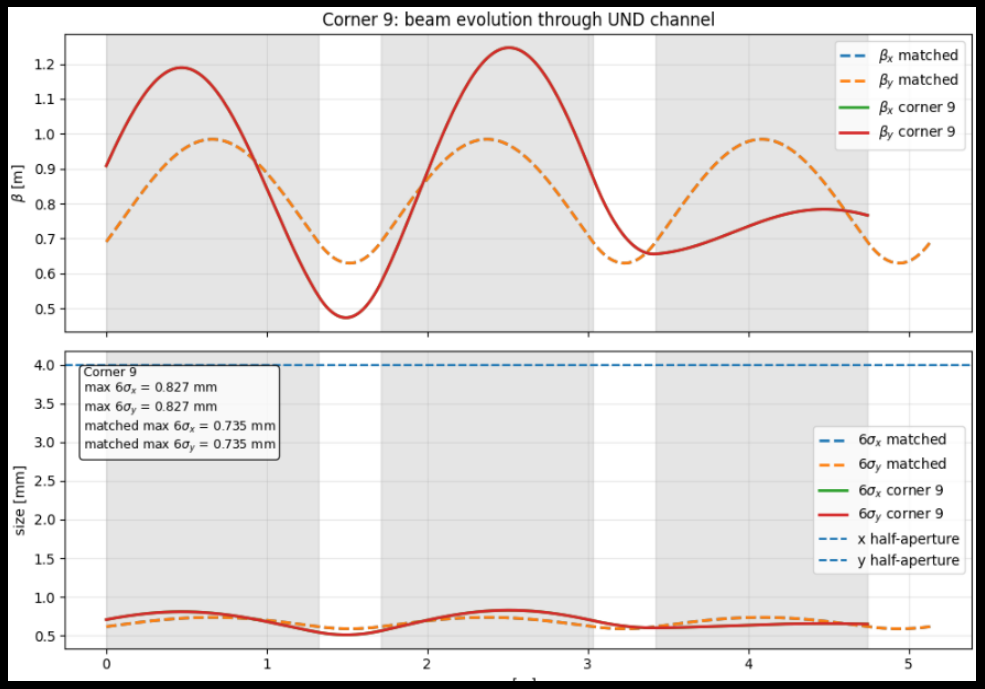
beta_y : 13.39013005990105 -> 29.868897977923364

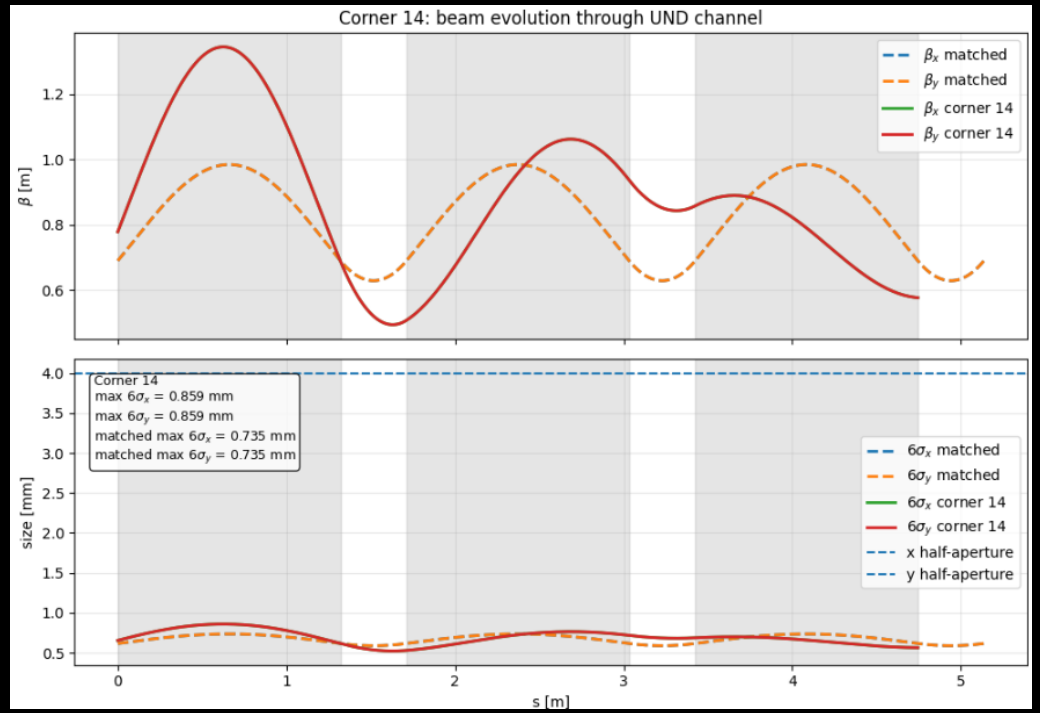


Corners index	αx Q1	βx Q1	αy Q1	βy Q1	α UND	β UND	Accepted	Max 6σ
0	-19.985	12.361	-21.873	13.390	-0.449	0.591	✅	0.815
1	-19.985	12.361	-21.873	29.869	0.001	0.617	✅	0.854
2	-19.985	12.361	-3.613	13.390	-0.661	0.779	✅	0.859
3	-19.985	12.361	-3.613	29.869	-0.399	0.539	✅	0.819
4	-19.985	24.121	-21.873	13.390	-0.611	0.695	✅	0.852
5	-19.985	24.121	-21.873	29.869	-0.336	0.526	✅	0.823
6	-19.985	24.121	-3.613	13.390	-0.661	0.779	✅	0.859
7	-19.985	24.121	-3.613	29.869	-0.561	0.669	✅	0.837
8	-2.814	12.361	-21.873	13.390	-0.661	0.779	✅	0.859
9	-2.814	12.361	-21.873	29.869	-0.511	0.908	✅	0.827
10	-2.814	12.361	-3.613	13.390	-0.661	0.779	✅	0.859
11	-2.814	12.361	-3.613	29.869	-0.449	0.591	✅	0.815
12	-2.814	24.121	-21.873	13.390	-0.661	0.779	✅	0.859
13	-2.814	24.121	-21.873	29.869	-0.561	0.669	✅	0.837
14	-2.814	24.121	-3.613	13.390	-0.661	0.779	✅	0.859
15	-2.814	24.121	-3.613	29.869	-0.611	0.695	✅	0.852


Corner 5: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 5
max 6σx = 0.823 mm
max 6σy = 0.823 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 5
βy corner 5
6σx matched
6σy matched
6σx corner 5
6σy corner 5
x half-aperture
y half-aperture


Corner 1: beam evolution through UND channel
β [m]
size [mm]
Corner 1
max 6σx = 0.854 mm
max 6σy = 0.854 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 1
βy corner 1
6σx matched
6σy matched
6σx corner 1
6σy corner 1
x half-aperture
y half-aperture


Corner 9: beam evolution through UND channel
β [m]
size [mm]
Corner 9
max 6σx = 0.827 mm
max 6σy = 0.827 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 9
βy corner 9
6σx matched
6σy matched
6σx corner 9
6σy corner 9
x half-aperture
y half-aperture


Corner 14: beam evolution through UND channel
β [m]
size [mm]
s [m]
Corner 14
max 6σx = 0.859 mm
max 6σy = 0.859 mm
matched max 6σx = 0.735 mm
matched max 6σy = 0.735 mm
βx matched
βy matched
βx corner 14
βy corner 14
6σx matched
6σy matched
6σx corner 14
6σy corner 14
x half-aperture
y half-aperture

$$x_{scaled} = x_{center} + s\,(x - x_{center})$$

Scale factor	Box enlargement	Failed corners	Surviving corners	Status
1.0	nominal safe box	0 / 16	16 / 16	fully safe
1.1	+10%	1 / 16	15 / 16	mostly stable
1.2	+20%	1 / 16	15 / 16	mostly stable
1.3	+30%	1 / 16	15 / 16	mostly stable
1.4	+40%	1 / 16	15 / 16	mostly stable
1.5	+50%	2 / 16	14 / 16	boundary degradation
1.6	+60%	3 / 16	13 / 16	boundary degradation
1.7	+70%	3 / 16	13 / 16	boundary degradation
1.8	+80%	3 / 16	13 / 16	boundary degradation
1.9	+90%	4 / 16	12 / 16	no longer uniformly safe
2.0	+100%	5 / 16	11 / 16	degraded